

One approach to model nonlinear weighting of probabilities

In the decision theory literature, e.g., rank-dependent models including (cumulative) prospect theory, it has been suggested that people overweight small probabilities when they are associated with the extreme outcomes.

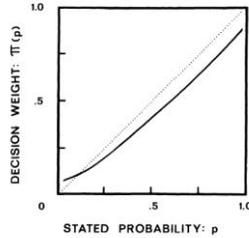


FIGURE 4.—A hypothetical weighting function.

Evidence of Dependence of Probability Weighting on Magnitude of Outcomes

- “Overweighting of small probabilities depends on the size of outcomes such that large outcomes engender greater curvature than smaller outcomes” (Camerer, 1992, cited in Tversky and Kahneman, 1992)
- People tend to be more pessimistic when facing large losses (Etchart-Vincent, 2004; Rottenstreich and Hsee, 2002)

Modeling Magnitude-Sensitive Probability Weighting

Start with EU

Consider a lottery which pays x_i with $1/N$ probability, we have:

$$EU = \sum v(x_i)/N.$$

- This does not work since the weights are constant.

Modeling Magnitude-Sensitive Probability Weighting

Consider nonconstant weights: e.g., assign $s_i/\sum s_i$ (instead of $1/N$) to the i th outcome x_i . The modified utility is then given by $\sum s_i v(x_i)/\sum s_i$. One way to make this expression symmetric is to limit its domain to the rank-ordered cone, e.g., when the outcomes are increasingly ordered, i.e., $x_1 \leq x_2 \leq \dots$. Notice that this underpins essentially all the rank-dependent models in the literature.

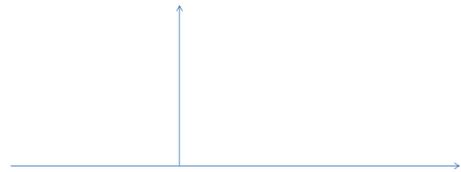
- Notice that this rank-dependent form can deliver overweighting, i.e., $s_i/\sum s_i > 1/N$, of the probability associated with a particular outcome, the degree of overweighting is not sensitive to its magnitude.

Modeling Magnitude-Sensitive Probability Weighting

Make the nonconstant weights magnitude dependent? A simple way is to let each nonconstant weight $s_i(x_i)$ depend on the corresponding outcome x_i . Doing this – introducing N additional functions of outcomes – is rather costly. A further simplification is to introduce one additional function s so that the decision weight assigned to outcome x_i is given by $s(x_i)/\sum s(x_i)$.

- Notice that this correspond the weighted utility function which is inherently symmetric.
- Moreover, the degree of overweighting $s(x_i)/\sum s(x_i)$ relative to $1/N$ is magnitude-sensitive but not rank-dependent.

Weighted (Salient) Utility: $\sum s(x_i)v(x_i)/\sum s(x_i)$

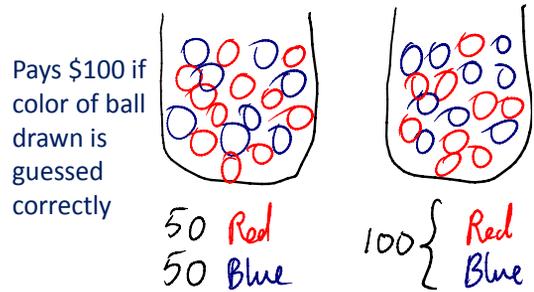


A U-shaped $s(x)$ can deliver

- overweighting of small probabilities tend to occur for sizable outcomes – both gains and losses
- dual model of reference dependence and loss aversion

More Paradoxical Evidence ...

Keynes' example appeared as Ellsberg's (1961) 2-urn paradox (cited in his dissertation)



Shanghai versus Dow Jones using Finance Professionals

Guess whether the trailing digit of a market index at closing the next business day is **odd** or **even**.

- A:  Reward: RMB260
- B:  Reward: RMB280

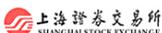
Which bet would you choose?

Dow Jones-Shanghai Study with Finance Professionals

- A:  Reward: RMB260 (18)
- B:  Reward: RMB280 (7)

Dow Jones-Shanghai Study with Finance Professionals

Guess whether a specific market index would be **up** or **down** at closing next business day?

- A:  Reward: RMB260
- B:  Reward: RMB280

Which bet would you choose?

Dow Jones-Shanghai Study with Finance Professionals

		
odd-even	RMB260 18	RMB 280 7
up-down	21	4

Keynes' insight of 1921

If two probabilities are equal in degree, ought we, in choosing our course of action, to prefer that one which is based on a **greater body of knowledge?**

A Treatise on Probability, 1921, Chapter VI

Antecedent ...

The typical case, ... , may be illustrated by the two cases following of balls drawn from an urn. ... ; in the first case we know that the urn contains black and white in equal proportions; in the second case the proportion of each colour is unknown,

Beijing-Tokyo Temperature Study

- Inspired by Fox-Tversky
- Part of 325-subject gene-brain-behavior study
- Odd or even of the temperature of a city

Beijing: RMB 11

Tokyo: RMB **13**



Beijing-Tokyo Temperature Study

- Part of 325-subject gene-brain-behavior study
- Odd or even of the temperature of a city

Beijing: RMB 11 (40%)

Tokyo: RMB **13 (60%)**



Equity Home Market Bias in the Lab

Individual Portfolio Choice

Cash points

	Stock Code	Points	Odd	Even
Stock 1	<input type="text"/>	<input type="text"/>	<input type="radio"/>	<input type="radio"/>
Stock 2	<input type="text"/>	<input type="text"/>	<input type="radio"/>	<input type="radio"/>
Stock 3	<input type="text"/>	<input type="text"/>	<input type="radio"/>	<input type="radio"/>

Note: At least 100 points for each stock chosen

Experimental Assets based on Trailing Digits of Closing Stocks Prices done at Max Planck

Stock	Return
L	2.5
P	2.5
V	2.5
W	2.5
F	2.7
H	2.7
I	2.7

- Endowed with 10000 points (10 Eur) + 2.5 Eur show-up fee
- Portfolio: Cash + up to 3 stocks
- Betting on odd/even of trailing digit of the closing price of the stocks chosen
- **WIN**: receive $R \times$ number of points invested in the stock

Demand for the 2 Kinds of Stocks

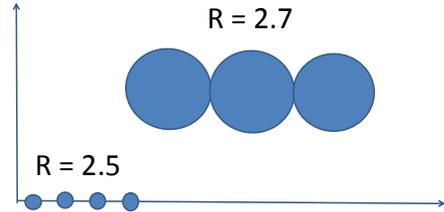


Table 1. List of Stocks Available for Forming the Portfolio

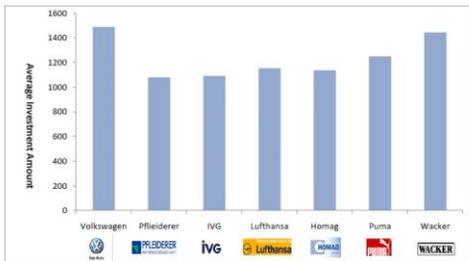
Logo	Company Name	Stock Code	R	Mean Familiarity (Std)
	Volkswagen AG St	239	2.5	9.5 (1.033)
	Pfleiderer AG	134	2.7	1.45 (1.545)
	IVG Immobilien AG	532	2.7	1.883 (1.688)
	Deutsche Lufthansa AG	342	2.5	9.067 (1.425)
	Homag Group AG	131	2.7	1.733 (1.821)
	Puma AG	332	2.5	9.117 (1.627)
	Wacker Chemie AG	423	2.5	3.55 (2.837)

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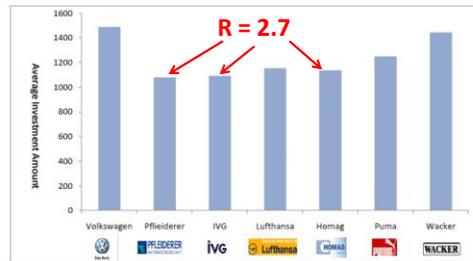
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Which has the highest demand ?

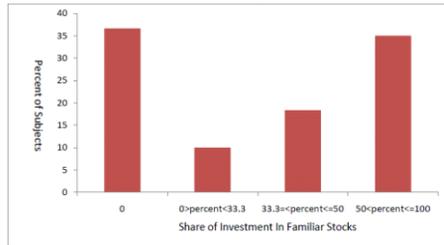
Observed Demand



Observed Demand



Share of High-Familiarity, Low-Return Stocks



Lesson from Observed Behavior

- Risk attitude may depend on the source of uncertainty
 - Can relate to underlying familiarity
- Equally likely events in terms of frequentist probability may **still** not be treated the same!

Modeling Affect Dependence

Incorporating additional affective aspects e of a lottery?

- e - appeal, beauty, noisy, crowded, ...
- We don't want to incorporate this into $v(x)$, so that $v(\$100, e) = v(\$100, e')$.
- Focus on possible dependence of probability weighting – always involves uncertainty – on affective aspects.
 - SU: $s(x; e) \rightarrow SU = \frac{\sum s(x_i, e)v(x_i)}{\sum s(x_i, e)}$

Modeling Ambiguity Aversion

- Modeling ambiguity aversion: In the 2-urn Ellsberg problem, the utility of betting on the known urn is given by

$$\frac{s(G, k)v(G)}{s(G, k) + 1}$$

- Utility of betting on the unknown urn is given by

$$\frac{s(G, u)v(G)}{s(G, u) + 1}$$

with $s(0, k) = s(0, u) = 1$. The latter is smaller than the former if we assume that “known” uncertainty is **more salient** than “unknown” uncertainty, i.e., s is **more salient than s^* if s/s^* is nondecreasing**.

34

Modeling Familiarity Bias

- To model familiarity bias, let k refer to the familiar and u refer to the unfamiliar in the previous slide.